

Spacecraft calorimetry as a test of the dark matter scattering model for flyby anomalies

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In previous papers we have shown that scattering of spacecraft nucleons from dark matter gravitationally bound to the earth gives a possible explanation of the flyby velocity anomalies. In addition to flyby velocity changes arising from the average over the scattering cross section of the collision-induced nucleon velocity change, there will be spacecraft temperature increases arising from the mean squared fluctuation of the collision-induced velocity change. We give here a quantitative treatment of this effect, and suggest that careful calorimetry on spacecraft traversing the region below 70,000 km where the flyby velocity changes take place could verify, or at a minimum place significant constraints, on the dark matter scattering model.

I. INTRODUCTION

In several recent papers we have explored the possibility that dark matter scattering may be responsible for the anomalous geocentric frame orbital energy changes that are observed during earth flybys of various spacecraft, as reported by Anderson et al. [1]. Some flybys show energy decreases, and others energy increases, with the largest anomalous velocity changes of order 1 part in 10^6 . While the possibility that these anomalies are artifacts of the orbital fitting method used in [1] is being actively studied, there is also a chance that they may represent new physics. In [2] we explored, through order of magnitude estimates, the possibility that the flyby anomalies result from the scattering of spacecraft nucleons from dark matter particles in orbit around the earth, with the observed velocity decreases arising from elastic scattering, and the observed velocity increases arising from exothermic inelastic scattering, which can impart an energy impulse to a spacecraft nucleon. In [3] we constructed a concrete model, based on two populations of dark matter particles, one of which scatters on nucleons elastically, and the other of which scatters inelastically, each with a shell-like distribution of orbits generated by the precession of a tilted circular orbit around the earth's rotation axis. We showed in [3] that this model can give a good fit to the flyby data, with

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shell radii in the 30,000–35,000 km range.

In the present paper we follow up on the brief observation in [2] that if there is a spacecraft velocity change as a result of dark matter scattering, there must be a corresponding temperature increase arising from fluctuations in the scattering recoil direction. In Sec. II we develop formulas for giving a quantitative treatment of this effect. In Sec. III we give order of magnitude quick estimates, and in Sec. IV we give numerical results based on the model of [3]. In Sec. V we suggest that a thermally shielded, spacecraft based calorimetry experiment could potentially give crucial information on the dark matter scattering model for the flyby anomalies.

II. TEMPERATURE CHANGE ARISING FROM VELOCITY FLUCTUATIONS

In [2] we considered the velocity change when a spacecraft nucleon of mass $m_1 \simeq 1\text{GeV}$ and initial velocity \vec{u}_1 scatters from a dark matter particle of mass m_2 and initial velocity \vec{u}_2 , into an outgoing nucleon of mass m_1 and velocity \vec{v}_1 , and an outgoing secondary dark matter particle of mass $m'_2 = m_2 - \Delta m$ and velocity \vec{v}_2 . (In the elastic scattering case, one has $m'_2 = m_2$ and $\Delta m = 0$.) Under the assumption that both initial particles are nonrelativistic, so that $|\vec{u}_1| \ll c, |\vec{u}_2| \ll c$, a straightforward calculation shows that the outgoing nucleon velocity is given by

$$\vec{v}_1 = \frac{m_1\vec{u}_1 + m_2\vec{u}_2}{m_1 + m'_2} + w\hat{v}_{\text{out}} \quad . \quad (1)$$

Here $w > 0$ is given¹ by taking the square root of

$$w^2 = \frac{m_2 m'_2}{(m_1 + m_2)(m_1 + m'_2)} (\vec{u}_1 - \vec{u}_2)^2 + \frac{\Delta m m'_2}{m_1(m_1 + m'_2)} \left[2c^2 - \frac{(m_1\vec{u}_1 + m_2\vec{u}_2)^2}{(m_1 + m_2)(m_1 + m'_2)} \right] \quad , \quad (2)$$

and \hat{v}_{out} is a kinematically free unit vector. Denoting by θ the angle between \hat{v}_{out} and the entrance channel center of mass nucleon velocity $\vec{u}_1 - (m_1\vec{u}_1 + m_2\vec{u}_2)/(m_1 + m_2) = m_2(\vec{u}_1 - \vec{u}_2)/(m_1 + m_2)$, and assuming that the center of mass scattering amplitude is a function $f(\theta)$ only of this polar angle, the average over scattering angles of the outgoing nucleon velocity is given by

$$\langle \vec{v}_1 \rangle = \frac{m_1\vec{u}_1 + m_2\vec{u}_2}{m_1 + m'_2} + w \langle \cos \theta \rangle \frac{\vec{u}_1 - \vec{u}_2}{|\vec{u}_1 - \vec{u}_2|} \quad , \quad (3)$$

with $\langle \cos \theta \rangle$ given by

$$\langle \cos \theta \rangle = \frac{\int_0^\pi d\theta \sin \theta \cos \theta |f(\theta)|^2}{\int_0^\pi d\theta \sin \theta |f(\theta)|^2} \quad . \quad (4)$$

¹ The notation t was used in [2] for what we here term w ; the change in notation avoids confusion with use of t for time. We take $\hbar = 1$ in Sec. III, while the velocity of light is denoted throughout by c .

Subtracting \vec{u}_1 from Eq. (3) gives the formula for the average velocity change used in [2] and [3] to calculate the flyby velocity change,

$$\langle \delta \vec{v}_1 \rangle = \frac{m_2 \vec{u}_2 - m'_2 \vec{u}_1}{m_1 + m'_2} + w \langle \cos \theta \rangle \frac{\vec{u}_1 - \vec{u}_2}{|\vec{u}_1 - \vec{u}_2|} . \quad (5)$$

However, in addition to contributing to an average change in the outgoing nucleon velocity, dark matter scattering will give rise to fluctuations in this velocity, which have a mean square magnitude given by

$$\langle (\vec{v}_1 - \langle \vec{v}_1 \rangle)^2 \rangle = w^2 \langle \left(\hat{v}_{\text{out}} - \langle \cos \theta \rangle \frac{\vec{u}_1 - \vec{u}_2}{|\vec{u}_1 - \vec{u}_2|} \right)^2 \rangle = w^2 (1 - \langle \cos \theta \rangle^2) . \quad (6)$$

This fluctuating velocity leads to an average temperature increase of the nucleon, per single scattering, of

$$\langle \delta T \rangle = \frac{m_1}{2k_B} \langle (\vec{v}_1 - \langle \vec{v}_1 \rangle)^2 \rangle = \frac{m_1}{2k_B} w^2 (1 - \langle \cos \theta \rangle^2) , \quad (7)$$

with k_B the Boltzmann constant. In analogy with the treatment of the velocity change $\delta \vec{v}_1$ in [2], to calculate dT/dt , the time rate of change of temperature of the spacecraft resulting from dark matter scatters, one multiplies the temperature change in a single scatter $\langle \delta T \rangle$ by the number of scatters per unit time. This latter is given by the flux $|\vec{u}_1 - \vec{u}_2|$, times the scattering cross section σ , times the dark matter spatial and velocity distribution $\rho(\vec{x}, \vec{u}_2)$. Integrating out the dark matter velocity, one thus gets for dT/dt at the point $\vec{x}(t)$ on the spacecraft trajectory with velocity $\vec{u}_1 = d\vec{x}(t)/dt$,

$$dT/dt = \int d^3 u_2 \langle \delta T \rangle |\vec{u}_1 - \vec{u}_2| \sigma \rho(\vec{x}, \vec{u}_2) . \quad (8)$$

Integrating from t_i to t_f we get for the temperature change resulting from dark matter collisions over the corresponding interval of the spacecraft trajectory ,

$$T_f - T_i = \int_{t_i}^{t_f} dt \int d^3 u_2 \langle \delta T \rangle |\vec{u}_1 - \vec{u}_2| \sigma \rho(\vec{x}, \vec{u}_2) . \quad (9)$$

In the elastic scattering case, with $\Delta m = 0$, $m'_2 = m_2$, the formula of Eq. (2) simplifies to

$$w^2 = \left(\frac{m_2}{m_1 + m_2} \right)^2 (\vec{u}_1 - \vec{u}_2)^2 . \quad (10)$$

In the inelastic case, assuming that $\Delta m/m_2$ and m'_2/m_2 are both of order unity, Eq. (2) is well approximated by

$$w^2 \simeq \left(\frac{2\Delta m m'_2}{m_1(m_1 + m'_2)} \right) c^2 . \quad (11)$$

Since \vec{u}_1 and \vec{u}_2 are typically of order 10 km s⁻¹, the temperature change in the inelastic case, per unit scattering cross section times angular factors, is larger than that in the elastic case by a factor $\sim c^2/|\vec{u}_1|^2 \sim 10^9$.

III. QUICK ESTIMATES

Before going on to detailed modeling calculations using Eq. (9), we first give quick estimates using Eqs. (7), (10), and (11), making the approximations that the dark matter mass m_2 is much smaller than the nucleon mass m_1 , and that $\langle \cos \theta \rangle$ in Eq. (7) is much smaller than 1. In the elastic case, Eq. (4) of [2] tells us that the magnitude of the velocity change in a single collision is of order

$$|\langle \delta \vec{v}_1 \rangle| \sim \frac{m_2}{m_1} |\vec{u}_1 - \vec{u}_2| \quad . \quad (12)$$

Taking the ratio of the single collision temperature change to the single collision velocity change, and multiplying by the flyby total velocity change $\sim 10^{-6} |\vec{u}_1|$, we get as an estimate of the total temperature change

$$T_f - T_i \sim \frac{\delta T}{|\langle \delta \vec{v}_1 \rangle|} 10^{-6} |\vec{u}_1| \sim 10^{-6} \frac{m_2}{2k_B} |\vec{u}_1| |\vec{u}_1 - \vec{u}_2| \sim 0.6 \times 10^{-5} \text{K} \left(\frac{m_2 c^2}{\text{MeV}} \right) \quad , \quad (13)$$

in agreement with [2].

In the inelastic case, we must take into account the kinematic structure of an exothermic inelastic differential cross section, as was done in [3] (but was not correctly done in the estimate given in [2]). In the inelastic case, Eq. (5) of [2] tells us that the magnitude of the velocity change in a single collision (for $\langle \cos \theta \rangle > 0$) is of order

$$|\langle \delta \vec{v}_1 \rangle| \sim \frac{\sqrt{2\Delta m m_2'}}{m_1} c \langle \cos \theta \rangle \quad . \quad (14)$$

Writing the inelastic differential cross section near threshold in the form

$$\frac{d\sigma}{d\Omega} = \frac{A_{\text{inel}}}{4\pi} k' k^{-1} + B_{\text{inel}} (k')^2 \frac{3}{4\pi} \cos \theta + \dots, \quad (15)$$

we have

$$\begin{aligned} \sigma &\simeq A_{\text{inel}} k' k^{-1} \quad , \\ \langle \cos \theta \rangle &\simeq B_{\text{inel}} k' / (A_{\text{inel}} k^{-1}) \quad , \end{aligned} \quad (16)$$

with k the entrance channel momentum

$$k = \frac{m_1 m_2}{m_1 + m_2} |\vec{u}_1 - \vec{u}_2| \simeq m_2 |\vec{u}_1 - \vec{u}_2| \quad , \quad (17)$$

and with k' the exit channel momentum, which to leading order in Δm is

$$k' \simeq \sqrt{2\Delta m m_2 c} \quad . \quad (18)$$

Again taking the ratio of the single collision temperature change to the single collision velocity change, and multiplying by the flyby total velocity change $\sim 10^{-6}|\vec{u}_1|$, we get as an estimate of the total temperature change in the inelastic case

$$T_f - T_i \sim \frac{\delta T}{|\langle \delta \vec{v}_1 \rangle|} 10^{-6} |\vec{u}_1| \sim \frac{10^{-6}}{2k_B} \frac{A_{\text{inel}}}{B_{\text{inel}}} \frac{|\vec{u}_1|}{|\vec{u}_1 - \vec{u}_2|} \frac{1}{m_2} \quad . \quad (19)$$

Defining a dimensionless parameter S_{inel} characterizing the inelastic scattering by

$$\frac{A_{\text{inel}}}{B_{\text{inel}}} \equiv (m_2 c)^2 S_{\text{inel}} \quad , \quad (20)$$

the estimate of Eq. (19) can be rewritten as

$$T_f - T_i \sim 10^{-6} \frac{m_2 c^2}{2k_B} S_{\text{inel}} \frac{|\vec{u}_1|}{|\vec{u}_1 - \vec{u}_2|} \sim 0.6 \times 10^4 \text{K} S_{\text{inel}} \left(\frac{m_2 c^2}{\text{MeV}} \right) \quad . \quad (21)$$

Thus if S_{inel} is of order unity, the inelastic scattering temperature rise is substantially bigger than that from elastic scattering, as already anticipated in the remarks following Eq. (11) above. For a dark matter-nucleon scattering force of range a , one expects $B_{\text{inel}}/A_{\text{inel}} \sim a^2$, giving the estimate $S_{\text{inel}} \sim 1/(m_2^2 c^2 a^2) = (\lambda_2/a)^2$, with $\lambda_2 = 1/(m_2 c)$ the dark matter Compton wavelength. So S_{inel} can be substantially less than unity.

Since $\langle \cos \theta \rangle \simeq k k' / (S_{\text{inel}} m_2^2 c^2)$, the approximation of neglecting $\langle \cos \theta \rangle$ compared to 1 is equivalent to assuming that $S_{\text{inel}} \gg k k' / (m_2 c)^2 = k k' \lambda_2^2$. We continue to make this assumption in the numerical work of the next section, and note that it is compatible with having a small value of S_{inel} .

IV. RESULTS FROM A MODEL FOR THE FLYBY VELOCITY ANOMALIES

In [3] we formulated a dark matter scattering model for the flyby velocity changes, by assuming that inelastic and elastic dark matter scatterers populate shells generated by the precession of circular orbits with normals tilted with respect to the earth's rotation axis. By some simple substitutions in the computer program used to calculate the velocity change predicted by a given set of model parameters, one can calculate the corresponding temperature increase of the spacecraft predicted by Eq. (9). Combining the elastic and inelastic scattering contributions, the results are conveniently written in the form

$$\frac{T_f - T_i}{^\circ\text{K}} = \left(\frac{m_2 c^2}{\text{MeV}} \right) (Z_{\text{el}} + S_{\text{inel}} Z_{\text{inel}}) \quad , \quad (22)$$

with Z_{el} and Z_{inel} dimensionless numbers giving respectively the elastic and inelastic contributions to the temperature increase. From fit 2d of [3], for which the elastic and inelastic shell radii are 29,370 km and 34,520 km respectively, we give the fits to the velocity anomalies and the corresponding values of Z_{el} and Z_{inel} in Table I. Except for Z_{el} for the NEAR spacecraft, which in fit 2d barely intersects the elastic shell, the values of Z_{el} and Z_{inel} are in accord with the quick estimates made in the preceding section.

V. DISCUSSION AND SUGGESTED EXPERIMENT

To summarize, if scattering from dark matter gravitationally bound to earth is responsible for the flyby velocity anomalies, there must also be spacecraft temperature increases when the spacecraft passes through the dark matter region. This suggests two space science investigations. The first is to analyze the records of both earth-orbiting satellites and earth-exiting spacecraft, to see if there are unexplained temperature anomalies, or at least to place bounds on temperature increases. The second is to design a dedicated, compact, thermally shielded calorimetry experiment that could be carried as a secondary payload on future space missions, to look for temperature increases as the spacecraft traverses the region within 70,000 km of the earth associated with the flyby velocity anomalies.

VI. ACKNOWLEDGEMENTS

This work was supported by the Department of Energy under grant DE-FG02-90ER40542.

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TABLE I: Flyby anomaly fit 2d and corresponding Z_{el} and Z_{inel} values.

	GLL-I	GLL-II	NEAR	Cassini	Rosetta	Messenger
δv_A (mm/s)	3.92	-4.6	13.46	-2	1.80	0.02
σ_A (mm/s)	0.3	1.0	0.01	1	0.03	0.01
δv_{th}	3.90	-4.6	13.46	-2.7	1.80	0.020
$Z_{\text{el}} \times 10^5$	0.26	0.85	0.17×10^{-3}	0.89	0.36	0.38
$Z_{\text{inel}} \times 10^{-4}$	0.37	0.36	0.74	0.20	0.65	0.58

The first two lines give the velocity discrepancy δv_A and the corresponding estimated error σ_A reported in [1]. The third line gives the theoretical values δv_{th} obtained from the model of [3], which has two shells of dark matter, one containing elastic scatterers, the other containing inelastic scatterers, gravitationally bound to the earth. The Z_{el} and Z_{inel} values, which give the flyby temperature rise when substituted in Eq. (22), are given in the final two lines. (In fit 2d, on which this table is based, the radius, Gaussian profile width, and tilt angle of the generating circular orbit are respectively 29,370 km, 6678 km, and 0.3902 radian for the elastically scattering shell, and 34,520 km, 3030 km, and 1.372 radian for the inelastically scattering shell.)